SOLUTION BY W. E. HEAL, WHEELING, INDIANA.

Let the indefinite integral

$$\int dx\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}=\varphi(x),\ \text{then}\int_0^x dx\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}=\varphi(x)\text{-}\varphi(0)=\frac{ny}{x}$$

Differentiating the above eq'n, (1), remembering that $\varphi(0)$ is constant,

$$dx\sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]}=\frac{n(xdy-ydx)}{x^2}. \tag{2}$$

Put $dy\div dx=p$, and (2) becomes $x^2\sqrt{[(1+p^2)]}=npx-ny$. (3)

Differentiating (3) we have

$$\frac{xpdp}{\sqrt{(1+p^2)}}+2dx\sqrt{(1+p^2)}=ndp. \tag{4}$$

Divide by $2(1+p^2)^{\frac{1}{4}}$; $dx(1+p^2)^{\frac{1}{4}}+xpdp\div(1+p^2)^{\frac{3}{4}}=ndp\div2(1+p^2)^{\frac{3}{4}}$. (5)

Integrating (5) and determining the constant $C=0$, we have

$x(1+p^2)^{\frac{1}{4}}=\frac{n}{2}\int\frac{dp}{(1+p^2)^{\frac{3}{4}}}$, (6). Let $p=\frac{2^{\frac{1}{2}}v(1-\frac{1}{2}v^2)^{\frac{1}{2}}}{1-v^2}$, then (6) becomes

$$\frac{x}{(1-v^2)^{\frac{1}{2}}}=\frac{n}{2^{\frac{1}{2}}}\int\frac{dv}{\sqrt{[(1-v^2)^3(1-\frac{1}{2}v^2)]}}=2^{\frac{1}{2}}n\left[\frac{v(1-\frac{1}{2}v^2)^{\frac{1}{2}}}{(1-v^2)^{\frac{1}{2}}}\right.$$

$$\left.+\tfrac{1}{2}\int\frac{dv}{(1-v^2)^{\frac{1}{2}}(1-\frac{1}{2}v^2)^{\frac{1}{2}}}-\int\frac{(1-\frac{1}{2}v^2)^{\frac{1}{2}}dv}{(1-v^2)^{\frac{1}{2}}}\right]=2^{\frac{1}{2}}n\left[\frac{v(1-\frac{1}{2}v^2)^{\frac{1}{2}}}{(1-v^2)^{\frac{1}{2}}}+\tfrac{1}{2}F(\tfrac{1}{2}2^{\frac{1}{2}},v)\right.$$

$\left.-E(\frac{1}{2}2^{\frac{1}{2}},v)\right]$ (7), where F and E are elliptic funct's of the 1st and 2nd ord's.

But $v=\frac{[(1+p^2)^{\frac{1}{2}}-1]^{\frac{1}{2}}}{(1+p^2)^{\frac{1}{2}}}$, and from (3) $p=\frac{n^2y+x\,[n^2(x^2+y^2)-x^4]}{x(n^2-x^2)}$.

Substituting these values in (7) we have the equation of the curve.

[The solution of 306, by Prof. Johnson, of 307, by Prof. Seitz, and of 308, by Mr. Adcock, will be published in No. 4.]

PROBLEMS.

309. *By Prof. Beman.*—In a given circle, find the vertices of the inscribed square, pentagon, octagon, and decagon by using the dividers alone.

310. *By Prof. Edmunds.*—Required the locus of vertices of a right angled spherical triangle whose legs pass through two fixed points given on the surface of the sphere.

311. *By Prof. Casey.*—A uniform circular plate is placed with its centre upon a prop, to find at what points on its circumference three given weights p, q, r must be attached that it may remain at rest in a horizontal position.

312. *By Prof. Scheffer.*—To find the area of the loop of the curve $y^3+x^2y-axy+bx^2=0$.